

## 5. Engineering economics

1. Introduction
2. Time value of money
3. Planning horizon and minimum attractive rate of return
4. Present worth analysis
5. Summary

### Text

White, Case, and Pratt, *Principles of Engineering Economic Analysis*, 5th ed., Wiley

## 5.1. Introduction

**Engineering economic analysis** is a combination of quantitative and qualitative techniques to analyze economic differences among engineering alternatives in selecting the preferred design.

The **cash flow approach** is one of the major approaches in the engineering economic analysis. A **cash flow** occurs when money actually changes hands from one individual to another or from organization to another.

- The individual events (transaction) of the cash flow (money received and money spent or paid) are distributed in time.
- The value of a given sum of money depends on both the amount of money and the point in time when the money is received or paid.

Any quantitative measure of the value of a given sum of money on both the amount of money and time of transaction can be called **time value of money (TVOF)**.

**Examples of TVOF:** Interest rate, discount rate, hurdle rate, minimum attractive rate of return, cost of capital.

The four simple rules allow one to reduce the transactions distributed in time to the same instant:

1. Money has a time value.
2. Money cannot be added or subtracted unless it occurs at the same point(s) in time
3. To move money forward one time unit, multiply by one plus the discount or interest rate
4. To move money backward one time unit, divide by one plus the discount or interest rate.

## 5.1. Introduction

The procedure of application of these rules is known as **discounted cash flow (DCF)**.

The DCF allows one to reduce all individual transaction to a single time instant. The overall sum of these transactions calculated in terms of the time value of money is called the **economic worth**. Different actual measures of the economic worth can be used depending on the problem.

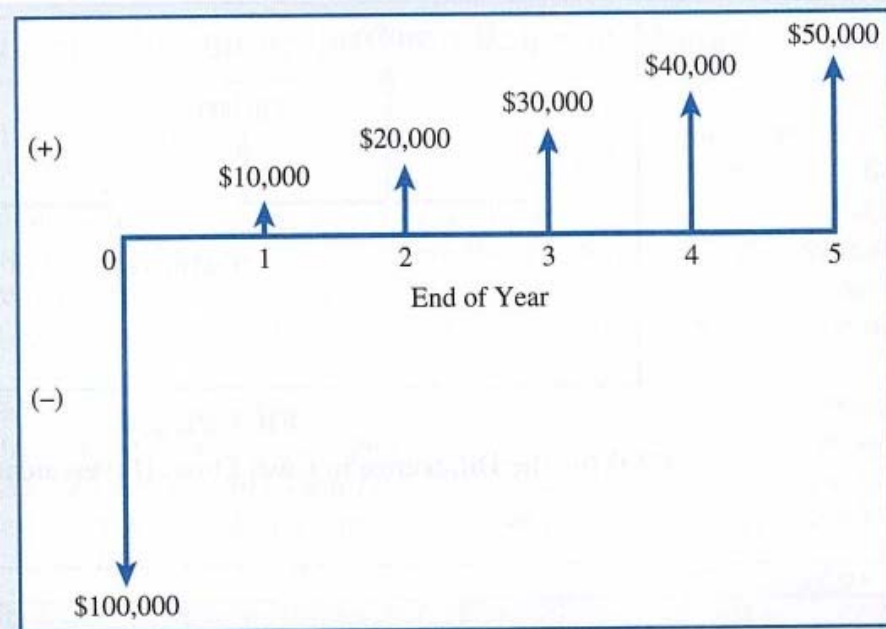
Roughly, the economic analysis implies calculation of the economic worth for all investment alternatives and choosing the option (or design) that the largest economic worth.

**The systematic economic analysis technique** implies solving the problem as a sequence of following steps

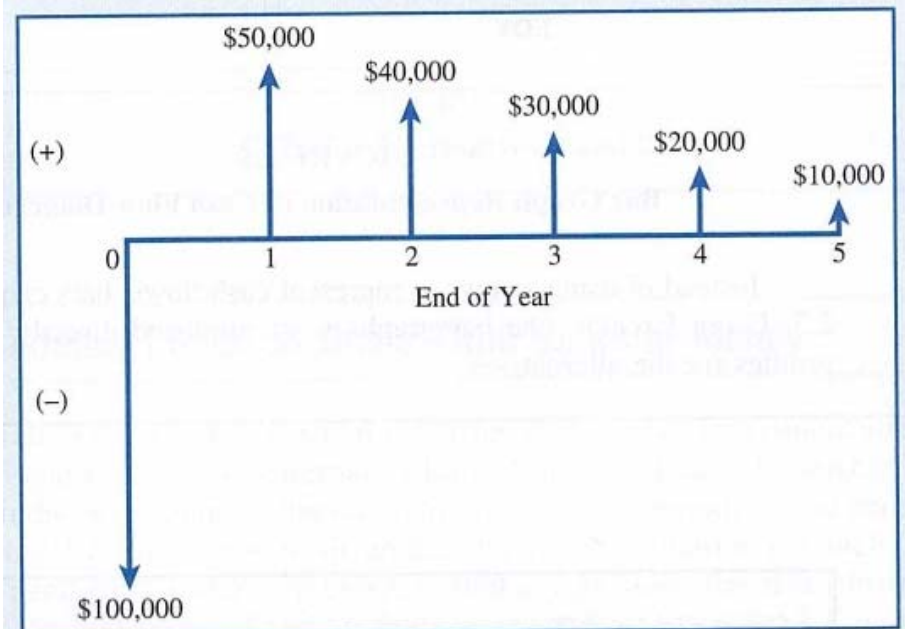
1. Identify the investment alternatives [What investment alternatives are available?]
2. Define the **planning horizon** [What is the length of time over which the decision is to be made?]
3. Specify the **discount rate** [What TVOM will be used to move money forward or backward in time?]
4. Estimate the cash flow [What are the best estimates of the cash flows for each of the alternatives?]
5. Compare the alternatives by calculating the economic worth [Which criterion seems best, based on the economic criterion chosen?]
6. Perform supplementary analysis [How sensitive is the economic preference to changes in or errors in the estimates used in the analysis?]
7. Select the preferred investment.

## 5.1. Introduction

Examples of the **cash flow diagrams** for two investments alternatives (see example 2.1 on page 38):



**FIGURE 2.2**  
*CFD for Investment A.*



**FIGURE 2.3**  
*CFD for Investment B.*

The alternative B is economically preferred.

Why?

In order to solve such problems we can use a principles of economic analysis, which are the experience-based rules for maximizing the profit.

## 5.0. Introduction

### Principles of Engineering Economic Analysis

1. Money has a time value;
2. Make investments that are economically justified;
3. Choose the mutually exclusive investment alternative that maximizes economic worth;
4. Two investment alternatives are equivalent if they have the same economic worth;
5. Marginal revenue must exceed marginal cost;
6. Money should continue to be invested as long as each additional increment of investment yields a return that is greater than the investor's time value of money;
7. Consider only differences in cash flows among investment alternatives;
8. Compare investment alternatives over a common period of time;
9. Risks and returns tend to be positively correlated; and
10. Past costs are irrelevant in engineering economic analyses, unless they impact future costs.

See the extended discussion of these principles in pages 6-8.

## 5.2. Time value of money

- Cash flow diagrams
- Simple interest calculations
- Compound interest calculations
- Simple/multiple cash flows
- Example: Determining the present worth of a gradient series

### Reading assignment

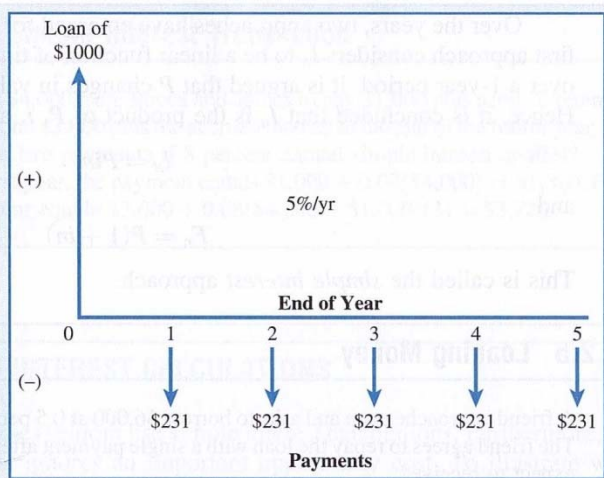
White, Case, and Pratt, 2.1-2.5

## 5.2. Time value of money

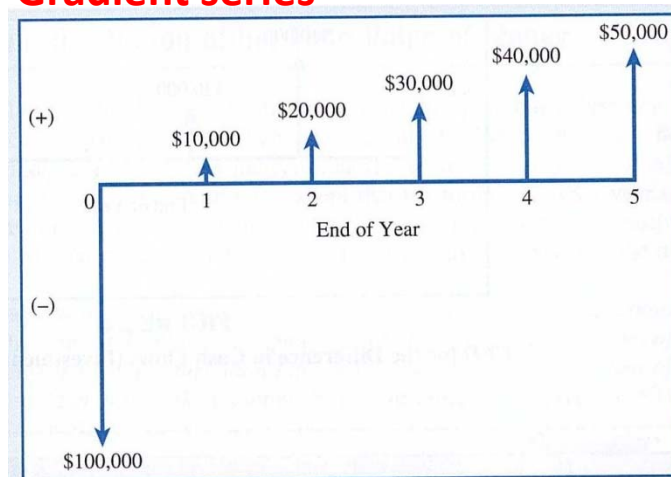
In this section, we consider how to calculate the time value of money (TVOF) at different times. The technical approaches for such calculations are based on three simple rules:

1. Money cannot be added or subtracted unless it occurs at the same point(s) in time
  2. To move money forward one time unit, multiply by one plus the discount or interest rate
  3. To move money backward one time unit, divide by one plus the discount or interest rate.
- Cash flow diagrams (CFDs) help to analyze the TVOF.
  - CFD shows received (+) and spent (-) money vs. time.
  - Two reasons to use CFD:
    - ✓ CFDs are powerful communication tools.
    - ✓ CFDs can help in identification of significant cash flow *patterns* within a sequence of economic transactions.

### Uniform series



### Gradient series





## 5.2. Time value of money

- Calculations of the time value of money is based on the establishing (postulating) a relationship between the present (current) value of a single sum of money and its future values after a single year or a fraction of an year.
- Such relationships are often postulated using the **interest rates**.

### Notation

$P$ , present value of money (value of money at a time when they are received or spent).

$n$ , number of years = planning horizon.

$F_n$ , future value of  $P$  after  $n$  years (accumulated value of  $P$  over  $n$  years).

$i$ , **annual interest rate**, i.e. the change in the value for \$1 over 1-year period.

$I_n = I_n(P, i, n)$ , **accumulated interest** in borrowing and lending transactions

$$F_n = P + I_n(P, i, n)$$

### Simple interest calculations

In simple interest calculations,  $I_n$  is assumed to be a linear function of time:

$$I_n = Pin, \quad F_n = P(1 + in) = F_{n-1} + Pi$$

### Compound interest calculations

In compound interest calculations, the interest rate  $i$  is the rate of change in the accumulated value of money

$$F_n = F_{n-1} + F_{n-1}i = F_{n-1}(1 + i) = F_{n-2}(1 + i)^2 = \dots = P(1 + i)^n$$



## 5.2. Time value of money

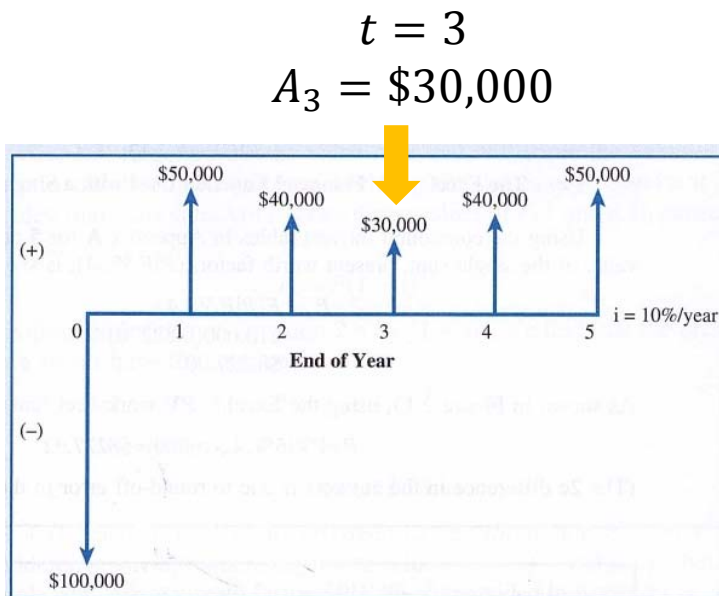
$$I_n = F_n - P = (F_n - F_{n-1}) + (F_{n-1} - P) = \dots = \sum_{t=1}^n iF_{t-1} \quad (F_0 = P)$$

These equations for  $F_n$  and  $I_n$  can be immediately applied to a **single cash flow**, i.e. to a single borrowing/lending transaction  $P$  made at time  $t = 0$ .

Now let's consider **multiple cash flow** with compound interest calculations:

We have multiple borrowing/lending of amounts of money  $A_t$  made at times  $t (= 0, 1, \dots, n)$ .

For a given planning horizon  $n$ , since these transactions are made at different times, we can be interested in both **present** ( $P = F_0$ ) and **future** ( $F = F_n$ ) **worth**.



For multiple cash flow, the total  $P$  or  $F$  accounts for *independent* contributions of all individual transaction with exponents  $n - t$  or  $-t$  that give the actual number of years between the time of transaction and present or future time.

$$P = A_0(1 + i)^0 + \dots + A_n(1 + i)^{-n} = \sum_{t=0}^n A_t(1 + i)^{-t}$$

$$F = A_0(1 + i)^n + \dots + A_n(1 + i)^0 = \sum_{t=0}^n A_t(1 + i)^{n-t}$$

## 5.2. Time value of money

### Example 1: Doubling your money; Rule of 72

How long does it take for an investment to double in value if it earns (a) 2 percent, (b) 3 percent, (c) 4 percent, (d) 6 percent, (e) 8 percent, or (f) 12 percent annual compound interest?

The first approach is to apply a rule of thumb called the *Rule of 72*. Specifically, the quotient of 72 and the interest rate provides a reasonably good approximation of the number of interest periods required to double the value of an investment:

- a.  $72/2 = 36$  yrs
- b.  $72/3 = 24$  yrs
- c.  $72/4 = 18$  yrs
- d.  $72/6 = 12$  yrs
- e.  $72/8 = 9$  yrs
- f.  $72/12 = 6$  yrs

With the third approach, we solve mathematically. Solving for  $n$  such that  $(1 + i)^n = 2$  gives  $n = \log 2 / \log(1 + i)$ . Therefore, the correct values of  $n$  (to 3 decimal places) are

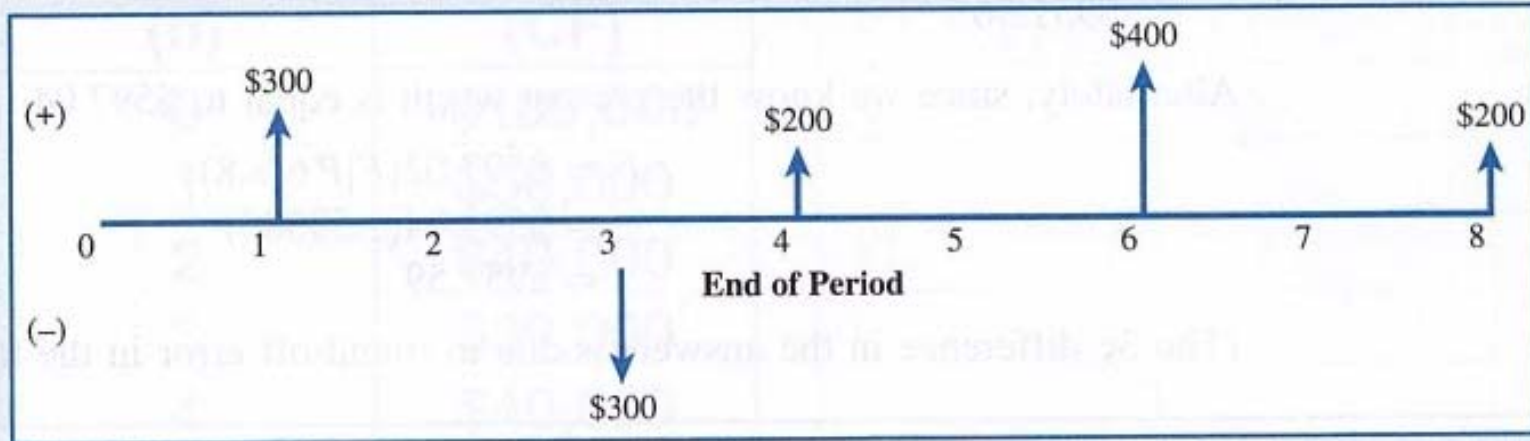
- a.  $n = \log 2 / \log 1.02 = 35.003$  yrs;
- b.  $n = \log 2 / \log 1.03 = 23.450$  yrs;
- c.  $n = \log 2 / \log 1.04 = 17.673$  yrs;
- d.  $n = \log 2 / \log 1.06 = 11.896$  yrs;
- e.  $n = \log 2 / \log 1.08 = 9.006$  yrs; and
- f.  $n = \log 2 / \log 1.12 = 6.116$  yrs.

## 5.2. Time value of money

**Example 2:** Computing the present worth of a series of cash flows

Consider the series of cash flows depicted by the *CFD* given in Figure 2.15. Using an interest rate of 6 percent per interest period, the present worth equivalent is given by

$$\begin{aligned} P &= \$300(P|F\ 6\%,1) - \$300(P|F\ 6\%,3) + \$200(P|F\ 6\%,4) + \$400(P|F\ 6\%,6) \\ &\quad + \$200(P|F\ 6\%,8) \\ &= \$300(0.94340) - \$300(0.83962) + \$200(0.79209) + \$400(0.70496) \\ &\quad + \$200(0.62741) \\ &= \$597.02 \end{aligned}$$



## 5.3. Planning horizon and minimum attractive rate of return

- Uncertainties in the engineering economic analysis
- Planning horizon
- Minimum attractive rate of return
- Weighted average cost of capital

### Reading assignment

White, Case, and Pratt, 4.1-4.4

## 5.3. Planning horizon and minimum attractive rate of return

Engineering economic analysis includes many uncertainties.

For instance, calculations of the present or future worth considered in section 6.1, require the following input data

- Cash flow events, individual transactions, their amounts and times.
- Planning horizon  $n$ .
- Some parameter like annual interest rate  $i$  that controls the time value of money (TVOF).

Usually decisions regarding the cash flow events (transactions) are quite definitive, since they are dictated by purchases of required materials and supplies, production circles, etc.

On the contrary, decisions regarding the other two parameters, planning horizon and TVOF, for many real-life problems include a lot of uncertainties. Decisions regarding these parameters are often made empirically, based on the experience.

In this section, we will consider semi-empirical rules that help us to make decisions regarding the choice of planning horizon and TVOF.

It is worth noting that due to uncertainties in input parameters, the best practice of the economic analysis requires not only *comparison of all economic alternatives* (e.g. different engineering designs), but also an *analysis of sensitivity of the results* (e.g., present or future worth) to the variation of the uncertain parameters (e.g.  $n$  or  $i$ ).



## 5.3. Planning horizon and minimum attractive rate of return

### Planning horizon

- Planning horizon, the length of time over which the decision is to be made, is usually counted in years.
- The choice of the planning horizon usually becomes non-trivial if different economic alternatives have unequal lives.
- In this case, there are six general approaches in determining the planning horizon length:

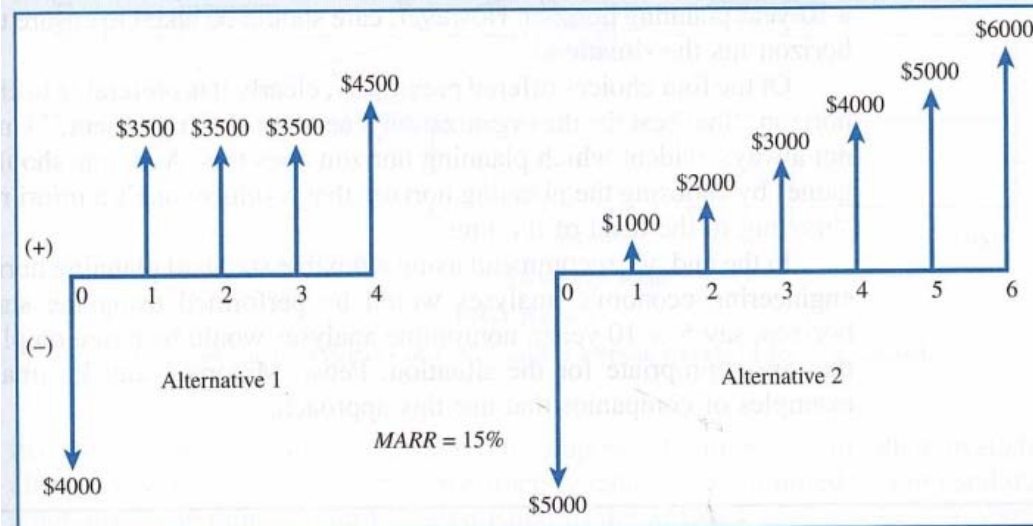
1. Set the planning horizon equal to the shortest life among the alternatives
2. Set the planning horizon equal to the longest life among the alternatives
3. Set the planning horizon equal to the least common multiple of the lives of the various alternatives
4. Use a standard length horizon, such as 10 years or 5 years
5. Set the planning horizon equal to the period of time that best fits the organization's need for the investment
6. Use an infinitely long planning horizon

The least common multiple of lives is a popular choice for the planning horizon's length. When it is used, the assumption is made that each alternative's cash flow profile repeats in the future until all investment alternatives under consideration conclude at the same time. Such an assumption might be practical for some applications but can prove untenable for others.

Two most popular choices

## 5.3. Planning horizon and minimum attractive rate of return

**Example:** See page 203



**FIGURE 4.5**  
CFDs for Example 4.2.

For this type of situation, a 6-year planning horizon is suggested, with zero cash flows occurring in years 5 and 6 with Alternative 1. At the end of 6 years, the net future worths for the two alternatives will be

$$\begin{aligned}
 FW_1(15\%) &= \$4,500(F|P\ 15\%,2) + \$3,500(P|A\ 15\%,3)(F|P\ 15\%,6) - \$4,000(F|P\ 15\%,6) \\
 &= \$4,500(1.32250) + \$3,500(2.28323)(2.31306) - \$4,000(2.31306) \\
 &= \$15,183.38 \\
 &= \mathbf{FV(15\%,6,-,PV(15\%,4,-3500,-1000)+4000) = \$15,183.34}
 \end{aligned}$$

$$\begin{aligned}
 FW_2(15\%) &= \$1,000(F|A\ 15\%,6) + \$1,000(A|G\ 15\%,6)(F|A\ 15\%,6) - \$5,000(F|P\ 15\%,6) \\
 &= \$1,000(8.75374) + \$1,000(2.09719)(8.75374) - \$5,000(2.31306) \\
 &= \$15,546.70 \\
 &= \mathbf{FV(15\%,6,-,NPV(15\%,1000,2000,3000,4000,5000,6000)+5000)} \\
 &= \mathbf{\$15,546.69}
 \end{aligned}$$

Thus, we would recommend Alternative 2.



## 5.3. Planning horizon and minimum attractive rate of return

### Minimum attractive rate of return

Calculations of TVOF use an interest rate to the compound (move forward in time) or discount (move backward in time) cash flows.

The question is: What is the minimum interest rate that makes any investment viable?

Such an interest rate we call the **minimum attractive rate of return (MARR)**.

The adopted value of MARR depends on many factors like type of the business etc. There are, however, some general rules that allows to roughly estimate MARR. These general rules are based on the fact that any investment will consume some portion of a firm's scarce resources and it is important for the investment to earn more than it costs to obtain the investment capital. It is important that

- MARR should be greater than the **cost of capital**, i.e. a firm's cost of obtaining capital.
- MARR should reflect the opportunity costs associated with investing in the candidate alternative as opposed to investing in other available alternatives.

Thus, the lower limit for the MARR is a firm's cost of capital.

The problem is that a company usually has available multiple source of capital: Loans, bonds, stocks, etc.

In order to estimate the MARR, we need a rule that allows us to reduce all these sources of the capital to single average value which is called the **weighted average cost of capital (WACC)**.

## 5.3. Planning horizon and minimum attractive rate of return

### Weighted average cost of capital

Capital available to a corporation can be categorized as either *debt capital* or *equity capital*. Examples of debt capital are bonds, loans, mortgages, and accounts payable; examples of equity capital are preferred stock, common stock, and retained earnings. Typically, capital for a particular investment consists of a mixture of debt and equity capital.

Although there are many variations available, a widely accepted WACC formula is

$$WACC = (E/V)i_e + (D/V)i_d(1 - itr) \quad (4.1)$$

where

$E$  = a firm's total equity, expressed in dollars

$D$  = a firm's total debt and leases, expressed in dollars

$V = E + D$ , a firm's total invested capital

$i_e$  = cost of equity or expected rate of return on equity, expressed in percent

$i_d$  = cost of debt or expected rate of return on borrowing, expressed in percent

$itr$  = corporate tax rate

- There are a lot of rules that allows one to estimate the individual contributions of various debts (loans, bonds, etc.) and equities (various stocks, etc.) to the firm's total debt and equity, see pages 207-212 (not necessary for the exam).

## 5.4. Present worth analysis

- Measures of economic worth
- Present worth: Single alternative
- Present worth: Multiple alternative
- Payback periods

### Reading assignment

White, Case, and Pratt, 5.1-5.7

## 5.4. Present worth analysis

There are nine methods to calculate the economic worth

1. The present worth (PW) method converts all cash flows to a single sum equivalent at time zero using  $i = MARR$ .
2. The future worth (FW) method converts all cash flows to a single sum equivalent at the end of the planning horizon using  $i = MARR$ .
3. The annual worth (AW) method converts all cash flows to an equivalent uniform annual series of cash flows over the planning horizon using  $i = MARR$ .
4. The internal rate of return (IRR) method determines the interest rate that yields a future worth (or present worth or annual worth) of zero.
5. The external rate of return (ERR) method determines the interest rate that equates the future worth of the invested capital to the future worth of recovered capital (when the latter is computed using the  $MARR$ .)
6. The modified internal rate of return (MIRR) method determines the interest rate that equates the present worth of invested capital (where the present worth is computed using a finance rate) to the future worth of recovered capital (where the future worth is computed using the  $MARR$ .)
7. The discounted payback period ( $DPBP$ ) method determines how long it takes for the cumulative present worth to be positive using  $i = MARR$ .
8. The capitalized worth (CW) determines the present worth (using  $i = MARR$ ) when the planning horizon is infinitely long.
9. The benefit/cost ratio (B/C) method determines the ratio of the present worth of benefits (savings or positive-valued cash flows) to the negative of the present worth of the investment(s) (or negative-valued cash flows) using  $i = MARR$ .

In this sections, we will consider only the present worth analysis



## 5.4. Present worth analysis

### Present worth: Single alternative

$$PW = \sum_{t=0}^n A_t(1 + MARR)^{-t}$$

**Example:** A single investment opportunity

To automatically insert electronic components in printed circuit boards for a cell phone production line, a \$500,000 surface mount placement (SMP) machine is being evaluated by a manufacturing engineer at Motorola. Over the 10-year planning horizon, it is estimated that the SMP machine will produce annual after-tax cost savings of \$92,500. The engineer estimates the machine will be worth \$50,000 at the end of the 10-year period. Based on the firm's 10 percent after-tax *MARR*, should the investment be made?

From our work in Chapter 2, we compute the present worth for the investment:

$$\begin{aligned} PW &= -\$500,000 + \$92,500(P|A 10\%, 10) + \$50,000(P|F 10\%, 10) \\ &= -\$500,000 + \$92,500(6.14457) + \$50,000(0.38554) \\ &= \$87,649.73 \end{aligned}$$

or, using Excel®,

$$\begin{aligned} PW &= PV(10\%, 10, -92500, -50000) - 500000 \\ &= \$87,649.62 \end{aligned}$$

Since  $PW > \$0$ , the investment is recommended.

With a present worth of **\$87,649.62**, there can be little doubt that the investment is a good one for the company. However, it is useful to examine how the present worth behaves over the 10-year period.

## 5.4. Present worth analysis

### Present worth: Multiple alternatives

**Ranking approach** (Applied when we are going to choose the preferred alternative among mutually exclusive alternatives): Choose the one with the greatest PW over planning horizon

$$\text{Maximize for all } j \quad PW_j = \sum_{t=0}^n A_{jt}(1 + MARR)^{-t}$$

### Example: Choosing between two alternatives

Entertainment Engineers, Inc., is an Ohio-based design engineering firm that, among other things, designs rides for amusement and theme parks all over the world. The rides range in complexity from very simple rides for young children to extremely sophisticated rides for adults. Entertainment Engineers employs aerospace, civil, electrical, industrial, materials, and mechanical engineers. Two alternative designs are under consideration for a new ride called the Scream Machine at a theme park located in Florida. The two candidate designs differ in complexity, cost, and predicted revenue. The first alternative design (A) will require an investment of \$300,000 and is estimated to produce after-tax revenue of \$55,000 annually over a 10-year planning horizon. The second alternative design (B) will require an investment of \$450,000 and is expected to generate annual after-tax revenue of \$80,000. A negligible salvage value is assumed for both designs. Theme park management could decide to “do nothing”; if so, the present worth of doing nothing will be zero. An after-tax MARR of 10 percent is used. Which alternative design, if either, should the theme park select?

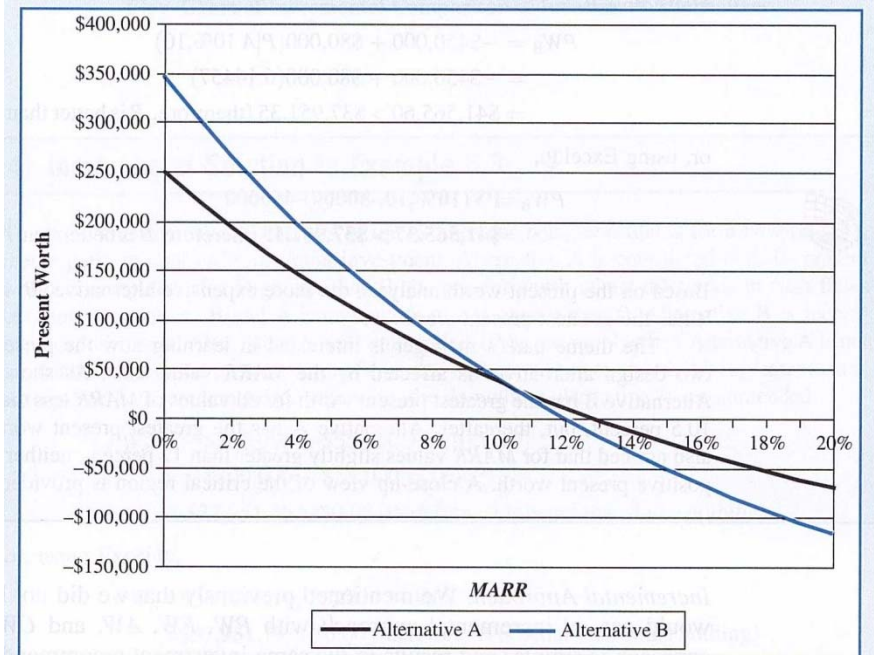
Letting A denote the alternative design for the \$300,000 initial investment and B denote the other alternative, the present worth of each is shown below:

#### Alternative A

$$\begin{aligned} PW_A &= -\$300,000 + \$55,000(P|A 10\%, 10) \\ &= -\$300,000 + \$55,000(6.14457) \\ &= \$37,951.35 > \$0.00 \text{ (therefore, A is better than doing nothing)} \end{aligned}$$

#### Alternative B

$$\begin{aligned} PW_B &= -\$450,000 + \$80,000(P|A 10\%, 10) \\ &= -\$450,000 + \$80,000(6.14457) \\ &= \$41,565.60 > \$37,951.35 \text{ (therefore, B is better than A)} \end{aligned}$$



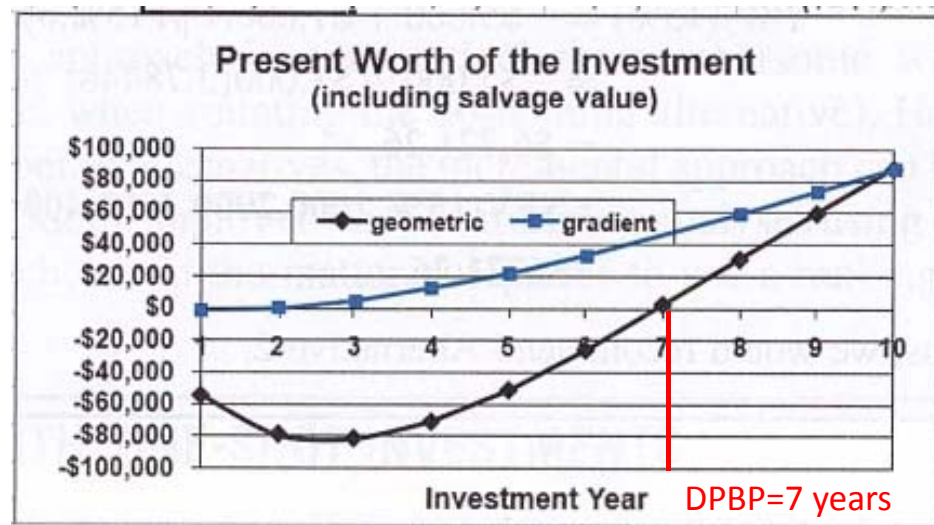


## 5.4. Present worth analysis

### Payback period

**Discounted payback period (DPBP)** for an investment is the time required for an investment to be fully recovered, including the time value of money, its the time required for a cumulative present worth to equal 0.

In order to calculate DPBP, we need to plot present worth vs. investment year. The year in which the cumulative present worth equals 0 is the DPBP.



**Payback period (PBP)** is the DPBP that does not incorporate the TVOM.

### Example:

For the \$500,000 investment in a surface mount placement machine that yields annual savings of \$92,500, the *PBP* is easily obtained:

$$PBP = \$500,000 / \$92,500 = 5.4054 \text{ years}$$



## 5.5. Summary

### For the exam we must know how

- To formulate the basic steps of the systematic economic analysis technique.
- To formulate the basic principles of the engineering economic analysis.
- To plot the cash flow diagrams.
- To choose the planning horizon and minimum attractive rate of return.
- To calculate the weighted average cost of capital
- To calculate the present and future worth for multiple cash flow and given compound annual interest rate.
- To use ranking approach in order to compare multiple investment alternatives.
- To give definitions of the discounted payback and payback periods.
- To calculate the payback period.